International Journal of Thermophysics, Vol. 25, No. 3, May 2004 (© 2004)

Reflection of Magneto-Thermoviscoelastic Waves under Generalized Thermoviscoelasticity

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Received June 13, 2003

Considering each of the thermal fields, viscoelastic and electromagnetic fields contribute to the total deformation of a body and interact with each other. Reflection of magneto-thermoelastic waves under generalized thermoviscoelastic theories is employed to study the reflection of plane harmonic waves from a semi-infinite elastic solid in a vacuum. The expressions for the reflection coefficients, which are the ratios of the amplitudes of the reflected waves to the amplitudes of the incident waves, are obtained, and the reflection coefficient ratios variation with the angle of incidence under different conditions are shown graphically when acrylic plastic materials are considered.

KEY WORDS: generalized magneto-thermoviscoelasticity; reflection coefficient ratio; rotational waves and dilatational waves; thermal relaxation times.

1. INTRODUCTION

With the rapid development of polymer science and the plastics industry, the use of materials under high temperature in modern technology, and the application of biology and geology in engineering, the study of the theory and application of viscoelastic materials has become an important subject in solid mechanics.

The theory of generalized thermoelasticity was proposed by Lord and Shulman [1] and Green and Lindsay [2] (hereafter called LS and GL theories, respectively). These theories have been developed by introducing one

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0195-928X/04/0500-0909/0

or two relaxation times in the thermoelastic process, with an aim to eliminate the paradox of infinite speed for the propagation of thermal signals. The LS model is based on a modified Fourier's law, but the GL model considers second sound even without violating the classical Fourier's law. The two theories are structurally different from one another, and one cannot be obtained as a particular case of the other. Various problems characterizing these two theories have been investigated and reveal some interesting phenomena. Brief reviews of this topic have been reported by Chandrasekharaiah [3, 4].

The theory of magneto-thermoelasticity is concerned with the interacting effects of the applied magnetic field on the elastic and thermoelastic deformations of a solid body. This theory has aroused much interest in many industrial applications, particularly in nuclear devices, where there exists a primary magnetic field; various investigations are to be carried out by considering the interactions among magnetic, thermal, and strain fields. Analyses of such problems also influence various applications in biomedical engineering as well as in different geomagnetic studies. The development of the interactions of an electromagnetic field, the thermal field, and the elastic field is available in many studies, such as Refs. 5–8.

In this paper, the generalized thermoviscoelastic theory is applied to study the reflection of a plane wave under a constant magnetic field for a semi-infinite viscoelastic solid nearby a vacuum. The variation of the reflection coefficient ratios of various reflected waves with the angle of incidence has been obtained for dynamical coupling theory, LS theory, and GL theory. Also, the effects of viscous and thermal coupling with the applied magnetic field are analyzed numerically.

2. FORMULATIONS OF THE PROBLEM

We consider an isotropic, homogeneous, linear, thermally and electrically conducting viscoelastic medium occupying the Cartesian semi-infinite space: $\Gamma=\{(x, y, z)| -\infty < x, y < \infty, -\infty < z \leqslant 0\}$. The whole body is at a constant temperature T_0 and it is acted on throughout by a constant magnetic field $\vec{\mathbf{H}}=(0, H_0, 0)$, which is oriented towards the positive direction of the y-axis.

The linearized equations of electromagnetism, valid for slowly moving media are as follows:

$$\vec{\mathbf{J}} = \operatorname{curl} \vec{\mathbf{h}} - \varepsilon_0 \dot{\vec{\mathbf{E}}}, \tag{1}$$

$$\operatorname{curl} \vec{\mathbf{E}} = -\mu_0 \dot{\vec{\mathbf{h}}}, \tag{2}$$

$$\vec{\mathbf{E}} = -\mu_0 \left(\dot{\vec{\mathbf{u}}} \wedge \vec{\mathbf{H}} \right), \tag{3}$$

$$\operatorname{div} \vec{\mathbf{h}} = 0. \tag{4}$$

The basic equation for magneto-thermoviscoelastic interactions in a homogeneous, isotropic solid in the context of coupled dynamical (CD) theory, LS theory, and GL theory may be taken in a unified form in the absence of body forces and heat sources:

(a) Equation of motion

$$\rho \ddot{u}_i = \sigma_{ij,j} + \mu_0 \left(\vec{\mathbf{J}} \times \vec{\mathbf{H}}\right)_i, \tag{5}$$

(b) Strain-displacement relation

$$\varepsilon_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right), \tag{6}$$

(c) Constitutive equation [9]

$$\sigma_{ij} = \int_0^t R(t-\tau) \frac{\partial \left[\varepsilon_{ij}\left(\vec{x}, \tau\right) - \alpha \hat{T}\left(\vec{x}, \tau\right)\right]}{\partial \tau}\, d\tau, \tag{7}$$

in which $\hat{T} = T + \tau_1 \dot{T}$, $R(t)$ is a relaxation function given by

$$R\left(t\right) = 2\mu \left[1 - A \int_0^t \mathrm{e}^{-\beta t} t^{\alpha^* - 1}\, dt\right], \tag{8}$$

and $(0 < \alpha^* < 1, A > 0, \beta > 0)$ are experimental parameters.

Assuming that the relaxation effects of the volume properties of the material are ignored, one can write for the generalized theory of thermoviscoelasticity,

$$\sigma = K\left[e - 3\alpha\left(T + \tau_1 \dot{T}\right)\right], \tag{9}$$

in which $\sigma = \sigma_{kk}/3$, $e = \varepsilon_{ii}$.

From Eqs. (7) and (9), we can obtain

$$\sigma_{ij} = \hat{R}\left(\varepsilon_{ij}\right) + \left(K - \frac{\hat{R}}{3}\right) e\delta_{ij} - \gamma\left(1 + \tau_1 \frac{\partial}{\partial t}\right) T\delta_{ij}, \tag{10}$$

in which

$$\int_0^t R(t-\tau) \frac{\partial \varepsilon_{ij}\left(\vec{x}, \tau\right)}{\partial \tau}\, d\tau = \hat{R}\left(\varepsilon_{ij}\right), \tag{11}$$

and δ_{ij} is the Kronecker Delta. The generalized thermoelasticity can be deduced from Eq. (10) by replacing $\hat{R}(\varepsilon_{ij})$ with $2\mu(\varepsilon_{ij})$.
(d) Heat conduction equation

$$k\nabla^2 T = \rho C_E \left(1 + \tau_0 \frac{\partial}{\partial t}\right)\dot{T} + T_0\gamma\left(1 + n\tau_0 \frac{\partial}{\partial t}\right)\mathrm{div}\dot{\vec{\mathrm{u}}}, \tag{12}$$

The use of relaxation times τ_0, τ_1 and dimensionless constant n makes the above equations possible for three different theories:

CD theory: $\tau_0 = \tau_1 = 0$
LS theory: $\tau_0 > 0, \quad \tau_1 = 0, \quad n = 1$
GL theory: $\tau_1 \geqslant \tau_0 \geqslant 0, \quad n = 0$

We shall consider only the two-dimensional problem. We assume that all variables are functions of space coordinates x, z, and time t and independent of coordinate y. So the displacement has components $\vec{u} = (u, 0, w)$.

Assume a perturbation of the magnetic field $\vec{h} = (0, \Omega, 0)$. To separate the dilatational and rotational components of strain, we introduce the displacement potentials ϕ and ψ by the relations,

$$u = \phi_{,\mathrm{x}} + \psi_{,\mathrm{z}}, \quad w = \phi_{,\mathrm{z}} - \psi_{,\mathrm{x}}. \tag{13}$$

From Eqs. (5), (7), (10), (12), and (14), we get

$$\alpha\frac{\partial^2\phi}{\partial t^2} = \left(\frac{K}{\rho} + \frac{2}{3\rho}\hat{R}\right)\nabla^2\phi - \frac{\gamma}{\rho}\left(1 + \tau_1\frac{\partial}{\partial t}\right)T - \frac{\mu_0 H_0}{\rho}\Omega, \tag{14}$$

$$\alpha\frac{\partial^2\psi}{\partial t^2} = \frac{1}{2\rho}\hat{R}\left(\nabla^2\psi\right), \tag{15}$$

$$k\nabla^2 T = \rho C_E\left(1 + \tau_0\frac{\partial}{\partial t}\right)\dot{T} + T_0\gamma\left(1 + n\tau_0\frac{\partial}{\partial t}\right)\nabla^2\dot{\phi}, \tag{16}$$

in which

$$\alpha = 1 + \frac{C_A^2}{c^2}, \quad c^2 = \frac{1}{\varepsilon_0\mu_0}, \quad C_A^2 = \frac{\mu_0 H_0^2}{\rho}.$$

we can obtain from Eqs. (1) to (4)

$$\Omega = -H_0\nabla^2\phi. \tag{17}$$

Let us introduce the following dimensionless variables:

$$\bar{x}_i = \frac{x_i}{C_T\omega_1}, \quad \bar{u}_i = \frac{u_i}{C_T\omega_1}, \quad \bar{T} = \frac{\gamma T}{K}, \quad \bar{\phi} = \frac{\phi}{(C_T\omega_1)^2}, \quad \bar{\psi} = \frac{\psi}{(C_T\omega_1)^2}, \tag{18}$$
$$\bar{\Omega} = \frac{\Omega}{H_0}, \quad \bar{t} = \frac{t}{\omega_1}, \quad \bar{\tau}_0 = \frac{\tau_0}{\omega_1}, \quad \bar{\tau}_1 = \frac{\tau_1}{\omega_1}, \quad \bar{\sigma}_{ij} = \frac{\sigma_{ij}}{K}, \quad \bar{\hat{R}} = \frac{2}{3k}\hat{R},$$
$$\omega_1 = \frac{k}{\rho c_E^2 C_T}, \quad C_T = \sqrt{\frac{K}{\rho}}, \quad C_L = \sqrt{\frac{\mu}{\rho}}, \quad R_H = \frac{C_A^2}{C_T^2}, \quad \bar{\varepsilon} = \frac{\gamma^2 T_0}{\rho^2 C_E {C_T}^2}.$$

In terms of these dimensionless variables, Eqs. (14) – (17) and Eqs. (8) and (10) take the following form (dropping the bar over the dimensionless variables for convenience):

$$\alpha\frac{\partial^2\phi}{\partial t^2} = \left(1 + \hat{R} + R_H\right)\nabla^2\phi - \left(1 + \tau_1\frac{\partial}{\partial t}\right)T, \tag{19}$$
$$\alpha\frac{\partial^2\psi}{\partial t^2} = \frac{3}{4}\hat{R}\left(\nabla^2\psi\right), \tag{20}$$
$$\nabla^2 T = \left(1 + \tau_0\frac{\partial}{\partial t}\right)\dot{T} + \varepsilon\left(1 + n\tau_0\frac{\partial}{\partial t}\right)\nabla^2\dot{\phi}, \tag{21}$$
$$\Omega = -\nabla^2\phi, \tag{22}$$
$$R(t) = \frac{4\mu}{3K}\left[1 - A\int_0^t \mathrm{e}^{-\beta t}t^{\alpha^*-1}\,dt\right], \tag{23}$$
$$\sigma_{ij} = \frac{3}{2}\hat{R}\left(\varepsilon_{ij}\right) + \left(1 - \frac{\hat{R}}{2}\right)e\delta_{ij} - \left(1 + \tau_1\frac{\partial}{\partial t}\right)T\delta_{ij}, \tag{24}$$

where R_H is the number of the magnetic pressure. This concept was originally introduced in magnetohydrodynamics. It is a measure of the relative importance of magnetic effects in comparison with mechanical ones. C_A is the so-called Alfvén speed. ε represents the usual thermoviscoelastic coupling parameters. Also, we can see from Eqs. (19)–(21) the dilatational waves are affected due to the presence of the thermal effect and magnetic field, while the coupled rotational waves remain unaffected.

3. SOLUTION OF THE PROBLEM

For a harmonic wave propagated in the direction that has the wave normal lie in the xz-plane,making an angle θ with the z-axis, we assume solutions of the system of Eqs. (19)–(22) in the forms:

$$\{\phi, T, \Omega\} = \{\phi_1, T_1, \Omega_1\}\exp\left[\mathrm{i}\{\xi\left(x\sin\theta + z\cos\theta\right) - \omega t\}\right], \tag{25}$$
$$\psi = \psi_1\exp\left[\mathrm{i}\{l\left(x\sin\theta + z\cos\theta\right) - \omega t\}\right],$$

where ξ, l, are the wave number and ω is the circular frequency.

Substituting Eq. (25) into Eqs. (19), (21), and (22), we arrive at a system of three homogeneous equations:

$$\left(\xi^2\beta_1-\alpha\omega^2\right)\phi_1+\tau_1' T_1=0, \tag{26}$$

$$\left(\xi^2-\mathrm{i}\omega\tau_0'\right)T_1+\mathrm{i}\omega\varepsilon\tau_0^*\xi^2\phi_1=0, \tag{27}$$

$$-\xi^2\phi_1+\Omega_1=0, \tag{28}$$

in which

$$\beta_1=1+R_H-\mathrm{i}\omega\bar{R},\quad \tau_0'=1-\mathrm{i}\omega\tau_0,\quad \tau_1'=1-\mathrm{i}\omega\tau_1,\quad \tau_0^*=1-\mathrm{i}\omega n\tau_0,$$
$$\bar{R}=-\frac{4\mu}{3K\mathrm{i}\omega}\left[1-\frac{A\Gamma(\alpha^*)}{(\beta-\mathrm{i}\omega)^{\alpha^*}}\right].$$

The system of Eqs. (26)–(28) has non-trivial solutions if and only if the determinant of the factor matrix vanishes. So

$$\begin{vmatrix}\xi^2\beta_1-\alpha\omega^2 & \tau_1' & 0\\ \mathrm{i}\omega\varepsilon\tau_0^*\xi^2 & \xi^2-\mathrm{i}\omega\tau_0 & 0\\ -\xi^2 & 0 & 1\end{vmatrix}=0. \tag{29}$$

This yields

$$\nu^4-\frac{\beta_1\tau_0'+\tau_0^*\tau_1'\varepsilon-\mathrm{i}\alpha\omega}{\alpha\tau_0'}\nu^2-\frac{\mathrm{i}\omega\beta_1}{\alpha\tau_0'}=0, \tag{30}$$

in which $\nu=\omega/\xi$ is the velocity of dilatational waves.

From Eqs. (22) and (25), we can obtain

$$w^2+\frac{3\mathrm{i}\omega\bar{\mathrm{R}}}{4\alpha}=0, \tag{31}$$

in which $w=\omega/l$ is the velocity of the rotational waves.

3.1. For Incident Rotational Wave

Since Eq. (30) is quadratic in ν^2, there shall be two dilatational waves traveling with two different velocities. So if assuming the radiation into vacuum is neglected, when a rotational wave falls on the boundary $z=0$ from within the viscoelastic medium, there shall be two reflected dilatational waves. We consider that the normal of the incident rotational

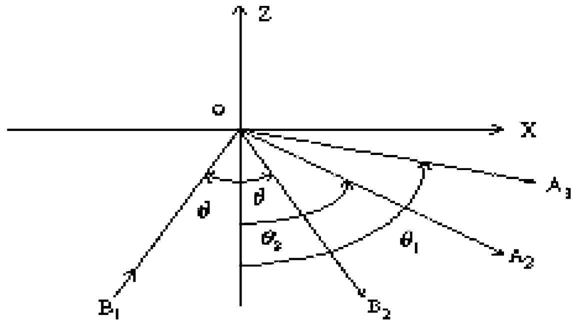


Fig. 1. Relation between the incident angle and the reflection angle.

waves makes an angle θ with the negative direction of the z axis, and the reflected dilatational waves will make angles θ_1, θ_2 with the same direction (Fig. 1), the displacement potentials ϕ and ψ will take the following forms:

$$\begin{aligned}\phi &= A_1 \exp\left[\mathrm{i}\left\{\xi_1\left(x\sin\theta_1 - z\cos\theta_1\right) - \omega t\right\}\right]\\ &\quad + A_2 \exp\left[\mathrm{i}\left\{\xi_2\left(x\sin\theta_2 - z\cos\theta_2\right) - \omega t\right\}\right], \qquad (32)\\ \psi &= B_1 \exp\left[\mathrm{i}\left\{l\left(x\sin\theta + z\cos\theta\right) - \omega t\right\}\right]\\ &\quad + B_2 \exp\left[\mathrm{i}\left\{l\left(x\sin\theta - z\cos\theta\right) - \omega t\right\}\right]. \qquad (33)\end{aligned}$$

The ratios of the amplitudes of the reflected waves and amplitudes of the incident waves $B_2/B_1, A_1/B_1, A_2/B_1$ give the corresponding reflection coefficients. Also it may be noted that the angles $\theta, \theta_1, \theta_2$ and the corresponding wave numbers l, ξ_1, ξ_2 are to be connected by the following relations:

$$\xi_1 \sin\theta_1 = \xi_2 \sin\theta_2 = l \sin\theta_0, \qquad (34)$$

on the interface z=0 of the medium. Equation (33) may also be written as

$$\frac{\sin\theta_1}{\nu_1} = \frac{\sin\theta_2}{\nu_2} = \frac{\sin\theta}{c'}, \qquad (35)$$

in which

$$\nu_1 = \frac{\omega}{\xi_1}, \quad \nu_2 = \frac{\omega}{\xi_2}, \quad c' = \frac{\omega}{l} = w = \sqrt{-\frac{3\mathrm{i}\omega\bar{R}}{4\alpha}} = \sqrt{-\frac{\beta_2}{\alpha}}, \qquad (36)$$

and v_1,v_2 are roots of Eq. (30).

3.2. For Incident Dilatational Wave

We consider that when the normal of the incident dilatational waves makes an angle θ with the negative direction of the z axis, there will be two reflected dilatational waves. One will make an angle θ with the direction of the z axis, and the other reflected dilatational wave and rotational wave will make angles θ_1, θ_2 with the same direction (Fig. 1); the displacement potentials ϕ and ψ will take the following forms:

$$\begin{aligned}\phi = {} & B_1 \exp\left[\mathrm{i}\left\{\xi_1 (x\sin\theta + z\cos\theta) - \omega t\right\}\right] \\ & + B_2 \exp\left[\mathrm{i}\left\{\xi_1 (x\sin\theta - z\cos\theta) - \omega t\right\}\right] \\ & + A_1 \exp\left[\mathrm{i}\left\{\xi_2 (x\sin\theta_1 - z\cos\theta_1) - \omega t\right\}\right], \end{aligned} \tag{37}$$

$$\psi = A_2 \exp\left[\mathrm{i}\left\{l (x\sin\theta_2 - z\cos\theta_2) - \omega t\right\}\right]. \tag{38}$$

Also, the angles $\theta, \theta_1, \theta_2$ and the corresponding wave numbers ξ_1, ξ_2, l are to be connected by the following relations:

$$\xi_1 \sin\theta = \xi_2 \sin\theta_1 = l \sin\theta_2, \tag{39}$$

on the interface z=0 of the medium. Equation (33) may also be written as

$$\frac{\sin\theta}{v_1} = \frac{\sin\theta_1}{v_2} = \frac{\sin\theta_2}{c'}, \tag{40}$$

in which v_1, v_2,and c' are the same as those obtained in Eq. (36).

4. BOUNDARY CONDITIONS

Since the boundary z=0 is adjacent to vacuum, it is free from surface traction. So the boundary condition can be expressed as

$$\sigma_{zj} = 0 \quad (j = x, y, z) \quad \text{for z=0.} \tag{41}$$

Assume the boundary z=0 is thermally insulated. This means that the following relation,

$$\frac{\partial T}{\partial z} = 0 \quad \text{for z=0.} \tag{42}$$

5. EXPRESSIONS FOR THE REFLECTION COEFFICIENTS

5.1. For Incident Rotational Wave

Using boundary conditions in Eqs. (41) and (42) and in Eqs. (32) and (33), we can obtain the following relations:

$$\frac{A_1}{B_1}\frac{c'^2}{\nu_1^2}\sin 2\theta_1+\frac{A_2}{B_1}\frac{c'^2}{\nu_2^2}\sin 2\theta_2-\frac{B_1}{B_2}\cos 2\theta-\cos 2\theta=0, \tag{43}$$

$$\frac{A_1c'^2}{B_1}\left(\frac{2}{\nu_1^2}\sin^2\theta_1-\frac{R_H}{\beta_2\nu_1^2}+\frac{\alpha}{\beta_2}\right)+\frac{A_2c'^2}{B_1}\left(\frac{2}{\nu_2^2}\sin^2\theta_2-\frac{R_H}{\beta_2\nu_2^2}+\frac{\alpha}{\beta_2}\right)$$
$$+\sin 2\theta-\frac{B_2}{B_1}\sin 2\theta=0, \tag{44}$$

$$\frac{A_1}{B_1}\left(\frac{\beta_1}{\nu_1^3}-\frac{\alpha}{\nu_1}\right)\cos\theta_1+\frac{A_2}{B_1}\left(\frac{\beta_1}{\nu_2^3}-\frac{\alpha}{\nu_2}\right)\cos\theta_2=0. \tag{45}$$

The solutions of this system for the reflection coefficients of rotational waves B_2/B_1, and the reflection coefficients of dilatational waves $A_1/B_1, A_2/B_1$ are

$$X1=\frac{A_1}{B_1}=\frac{P_1}{Q},\quad X2=\frac{A_2}{B_1}=\frac{P_2}{Q},\quad X3=\frac{B_2}{B_1}=\frac{P_3}{Q}, \tag{46}$$

for which

$$P_1=2\beta_2\nu_1^3\left(\beta_1-\alpha\nu_2^2\right)\cos\theta_2\sin 2\theta\cos 2\theta, \tag{47}$$

$$P_2=2\beta_2\nu_2^3\left(\beta_1-\alpha\nu_1^2\right)\cos\theta_1\sin 2\theta\cos 2\theta, \tag{48}$$

$$P_3=-\nu_1c'^2\left(\beta_1-\alpha\nu_2^2\right)\cos\theta_2\left[\left(\alpha\nu_1^2-R_H+\beta_2\right)\cos 2\theta-\beta_2\cos 2\left(\theta+\theta_1\right)\right]$$
$$+\nu_2c'^2\left(\beta_1-\alpha\nu_1^2\right)\cos\theta_1\left[\left(\alpha\nu_2^2-R_H+\beta_2\right)\cos 2\theta-\beta_2\cos 2\left(\theta+\theta_1\right)\right] \tag{49}$$

$$Q=\nu_1c'^2\left(\beta_1-\alpha\nu_2^2\right)\cos\theta_2\left[\left(\alpha\nu_1^2-R_H+\beta_2\right)\cos 2\theta-\beta_2\cos 2\left(\theta-\theta_1\right)\right]$$
$$-\nu_2c'^2\left(\beta_1-\alpha\nu_1^2\right)\cos\theta_1\left[\left(\alpha\nu_2^2-R_H+\beta_2\right)\cos 2\theta-\beta_2\cos 2\left(\theta-\theta_1\right)\right]. \tag{50}$$

5.2. For Incident Dilatational Wave

Using boundary conditions (41) and (42) and Eqs. (37) and (38), we can obtain the following relations:

$$\sin 2\theta - \frac{B_2}{B_1}\sin 2\theta - \frac{A_1}{B_1}\frac{\nu_1^2}{\nu_2^2}\sin 2\theta_1 + \frac{A_1}{B_1}\frac{\nu_1^2}{c'^2}\cos 2\theta_2 = 0, \tag{51}$$

$$1 + \frac{B_2}{B_1} + \frac{A_1\nu_1^2}{B_1\nu_2^2}\frac{2\beta_2\sin^2\theta_1 - R_H + \alpha\nu_2^2}{2\beta_2\sin^2\theta - R_H + \alpha\nu_1^2} + \frac{A_2\nu_1^2}{B_1 c'^2}\frac{\beta_2\sin 2\theta_2}{2\beta_2\sin^2\theta - R_H + \alpha\nu_1^2} = 0 \tag{52}$$

$$\cos\theta - \frac{B_2}{B_1}\cos\theta - \frac{A_1\nu_1^3}{B_1\nu_2^3}\frac{\alpha\nu_2^2 - \beta_1}{\alpha\nu_1^2 - \beta_1}\cos\theta_1 = 0. \tag{53}$$

The solutions of this system for the reflection coefficients of dilatational waves B_2/B_1 and A_1/B_1, and the reflection coefficients of rotational waves A_2/B_1, are

$$X1 = \frac{B_2}{B_1} = \frac{R_1}{W}, \quad X2 = \frac{A_1}{B_1} = \frac{R_2}{W}, \quad X3 = \frac{A_2}{B_1} = \frac{R_3}{W}, \tag{54}$$

in which

$$\begin{aligned} R_1 = {} & -\nu_1^3(\alpha\nu_2^2 - \beta_1)\cos\theta_1[(R_H - \alpha\nu_1^2)\cos 2\theta_2 - 2\beta_2\sin\theta\sin(\theta - 2\theta_2)] \\ & -\nu_2\nu_1^2(\alpha\nu_1^2 - \beta_1)\cos\theta[(R_H - \alpha\nu_2^2)\cos 2\theta_2 - 2\beta_2\sin\theta_1\sin(\theta_1 + 2\theta_2)], \end{aligned} \tag{55}$$

$$R_2 = -2\nu_2^3(\beta_1 - \alpha\nu_1^2)(R_H - \alpha\nu_1^2 - 2\beta_2\sin^2\theta)\sin 2\theta\cos 2\theta_2, \tag{56}$$

$$\begin{aligned} R_3 = {} & -2c'^2\left[\nu_1(\beta_1 - \alpha\nu_2^2)\sin 2\theta\cos\theta_1 - \nu_2(\beta_1 - \alpha\nu_1^2)\sin 2\theta_1\cos\theta\right] \\ & \times (R_H - \alpha\nu_1^2 - 2\beta_2\sin^2\theta), \end{aligned} \tag{57}$$

$$\begin{aligned} W = {} & \nu_1^2 - \left\{\nu_1\left(\beta_1 - \alpha\nu_2^2\right)\cos\theta_1\left[\left(R_H - \alpha\nu_1^2\right)\cos 2\theta_2 \right.\right. \\ & \left. -2\beta_2\sin\theta\sin(\theta + 2\theta_2)\right] - \nu_2\left(\beta_1 - \alpha\nu_1^2\right) \\ & \left.\times \cos\theta\left[\left(R_H - \alpha\nu_2^2\right)\cos 2\theta_2 + 2\beta_2\sin\theta_1\sin(\theta_1 + 2\theta_2)\right]\right\}, \end{aligned} \tag{58}$$

6. SPECIAL CASES

6.1. In the Absence of Thermal Coupling

In this case, we set $\varepsilon = 0$. From Eq. (30) we can obtain

$$\nu_1^2 = \beta_1/\alpha, \quad \nu_2^2 = -\mathrm{i}\omega/\tau_0', \tag{59}$$

and the velocities of the wave are given by

$$\nu_1 = \sqrt{\frac{\beta_1}{\alpha}}, \quad \nu_2 = \mathrm{i}^{\frac{3}{2}}(\omega/\tau_0')^{\frac{1}{2}}. \tag{60}$$

6.1.1. For Incident Rotational Wave

For an incident rotational wave, we can obtain the expressions for $X1$, *X2,* and *X3*:

$$X1 = \frac{n_1}{n}, \quad X2 = 0, \quad X3 = \frac{n_3}{n}, \tag{61}$$

for which

$$n_1 = 2\beta_1 \sin 2\theta \cos 2\theta, \tag{62}$$

$$n_3 = (\beta_1 - R_H + \beta_2)\cos 2\theta - \beta_2 \cos 2(\theta + \theta_1), \tag{63}$$

$$n = -(\beta_1 - R_H + \beta_2)\cos 2\theta + \beta_2 + \cos 2(\theta + \theta_1). \tag{64}$$

6.1.2. For an Incident Dilatational Wave

For an incident dilatational wave we can obtain the expressions for $X1, X2,$and $X3$:

$$X1 = \frac{m_1}{m}, \quad X2 = 0, \quad X3 = \frac{m_3}{m}, \tag{65}$$

$$R_1 = -\nu_1^2[(R_H - \beta_1)\cos 2\theta_2 - 2\beta_2 \sin\theta \sin(\theta - 2\theta_2)], \tag{66}$$

$$R_3 = 2c'^2 \sin 2\theta (R_H - \beta_1 - 2\beta_2 \sin^2\theta), \tag{67}$$

$$W = -\nu_1^2[(R_H - \beta_1)\cos 2\theta_2 - 2\beta_2 \sin\theta \sin(\theta + 2\theta_2)]. \tag{68}$$

From Eqs. (61) and (65) we can clearly see that if an incident wave arrived at the boundary plane, there will be two reflected waves; one is the reflected rotational wave *X3* and the other is the reflected dilatational wave *X1*. Also, we can see that the relaxation times have no effects on the reflection coefficients. So the CD theory, LS theory, and GL theory give the same result when the thermal coupling is not considered.

6.2. In the Absence of Viscous Effects

The generalized thermoelasticity can be deduced from Eq. (10) by replacing $\hat{R}(\varepsilon_{ij})$ by $2\mu(\varepsilon_{ij})$; in this case, the constants β_1, β_2 and the velocity of the rotational wave are given below:

$$\beta_1 = 1 + R_H, \quad \beta_2 = \sqrt{\frac{\mu}{\lambda + 2\mu}}, \quad c' = \sqrt{\frac{\beta_2}{\alpha}} \tag{69}$$

We can obtain the reflection coefficient ratios by substituting β_1, β_2,c' in Eqs. (47)–(50) and Eqs. (55)–(58) by Eq. (69).

7. NUMERICAL RESULTS

An acrylic plastic material was chosen for the purpose of numerical evaluation. For the plastic Poisson's ratio can be taken equal to 0.25 which leads to $4\mu/3K = 0.8$. The constants of the problem were taken as [9]: $\beta = 0.005$, $A = 0.106$, $\alpha^* = 0.5$, $\tau_0 = 1.0$, $\tau_1 = 1.2$.

Figs. 2 and 3 give the variation of the reflection coefficient ratios with the angle of incidence for a rotational wave and a dilatational wave under three theories. We can see that in the case of a rotational wave, the reflection coefficient ratio $|X1| = |X2| = 0$ when $\theta = 45^\circ$. Figs. 4 and 5 give a comparison of the reflection coefficient ratios between the LS case with and without the viscous effect, and the GL case with and without the viscous effect. It can be observed that the viscous effect plays an important role. Also, we can see that the viscous effect is more pronounced in the GL theory than in the LS theory. Figs. 6 and 7 give the variation of the angle of incidence with the reflection coefficient ratios under different values of coupling parameters. We can see that the reflection coefficient ratios $|X1|, |X3|$ decrease with an increase of the coupling parameter, while the reflection coefficient ratio $|X2|$ increases with an increase of the coupling parameter. Also, it can be observed that $|X2|$ vanishes when $\varepsilon = 0$ as that discussed in a particular case. Figs. 8 and 9 give the effect of a magnetic field on the reflection coefficient ratio. Clearly the magnetic field has a salient influence on the reflection coefficient ratio. Also, we can see that the effect of the magnetic field is more important in the case of dilatational incidence than in the case of rotational incidence.

8. CONCLUSIONS

We can obtain the following conclusions according to the above analysis:

1. The reflection coefficient ratio depends on the angle of incidence; the nature of this dependence is different for different reflected waves.

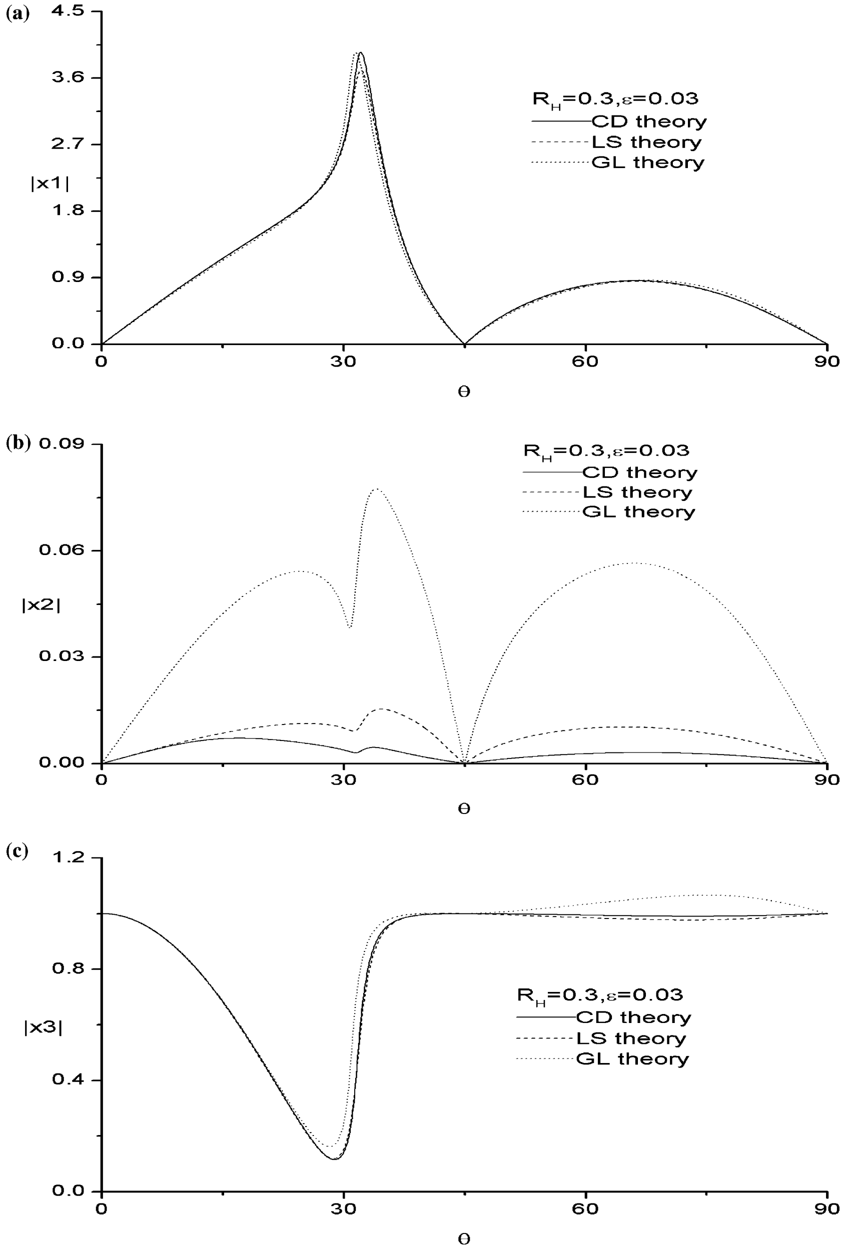
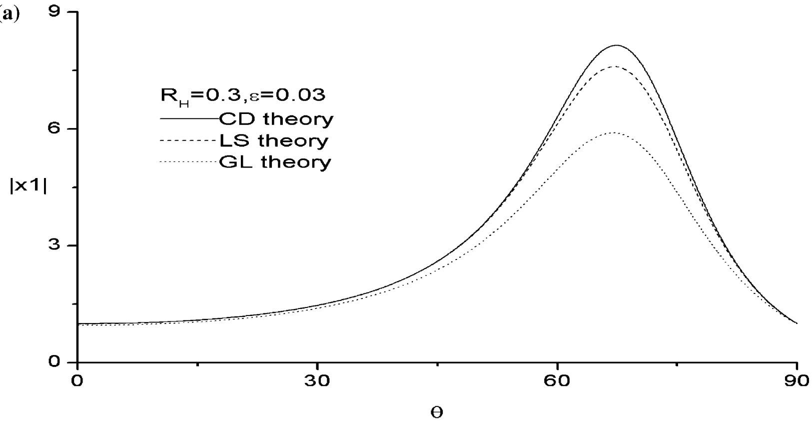
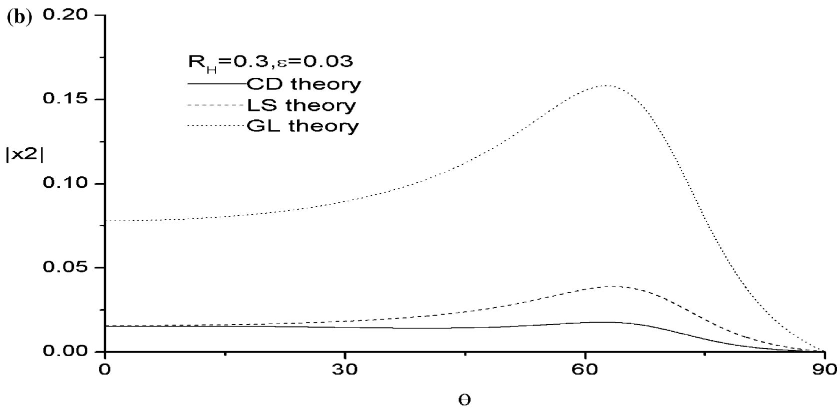


Fig. 2. Variation of reflection coefficient ratio with incident angle of rotational wave under different theories.

(a)



(b)



(c)

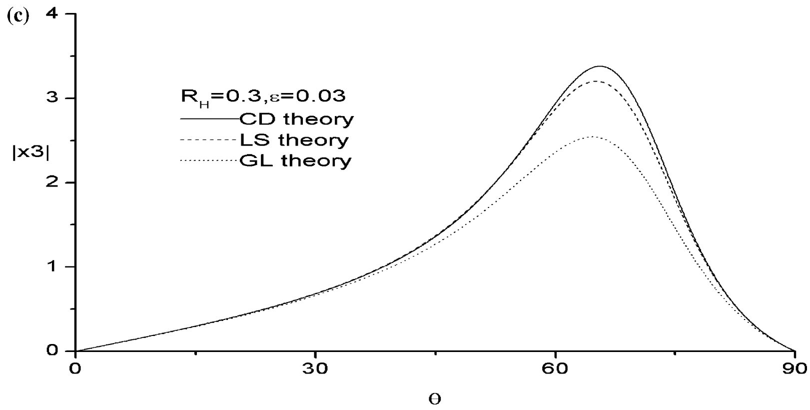


Fig. 3. Variation of reflection coefficient ratio with incident angle of dilatational wave under different theories.

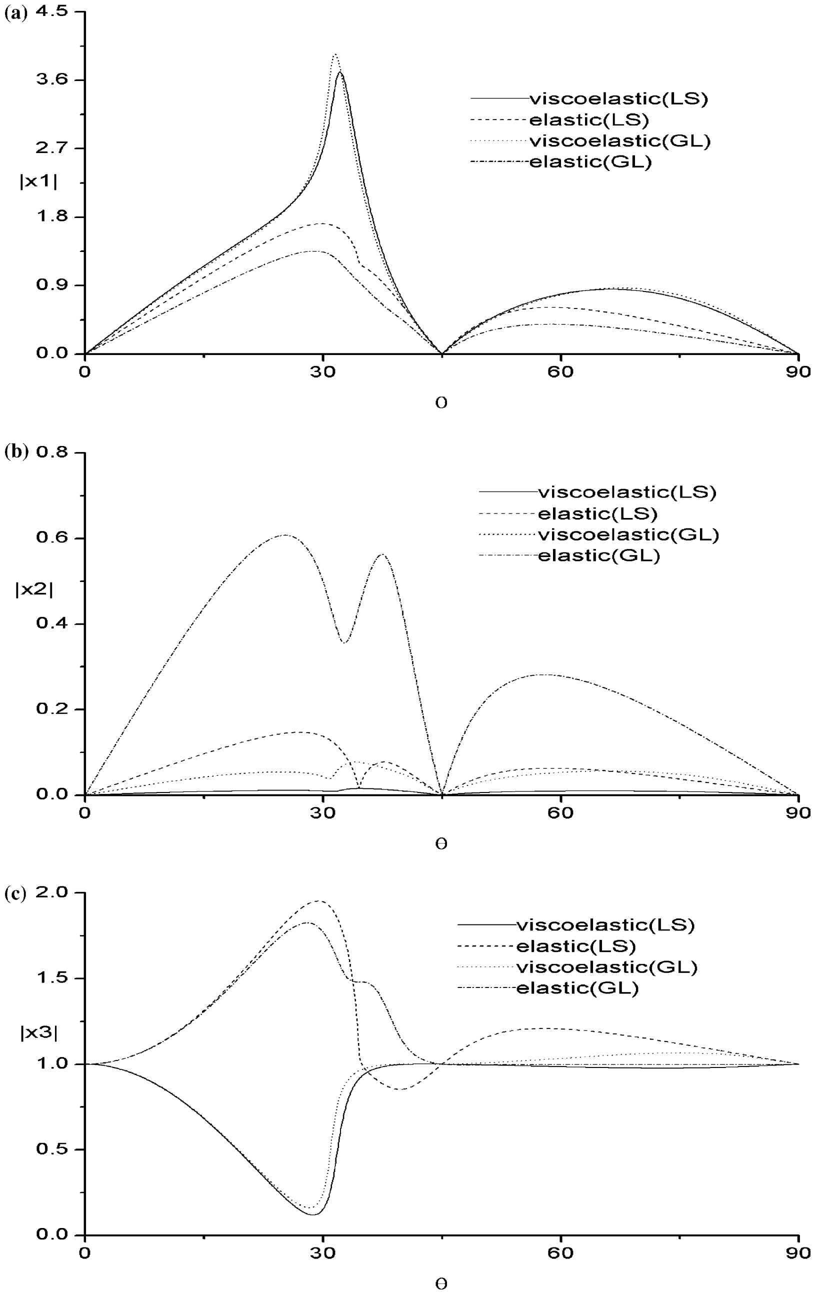
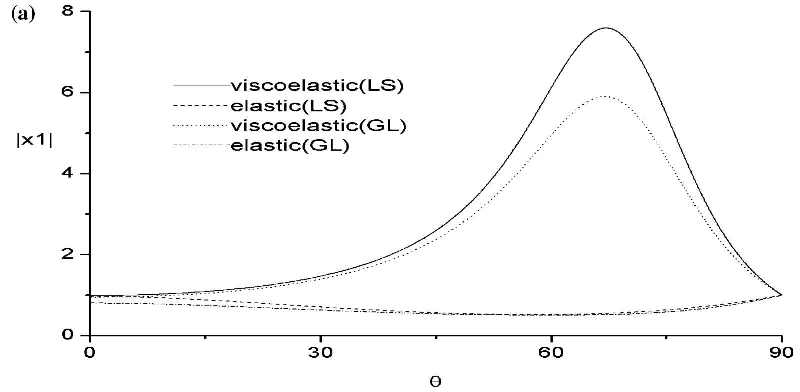
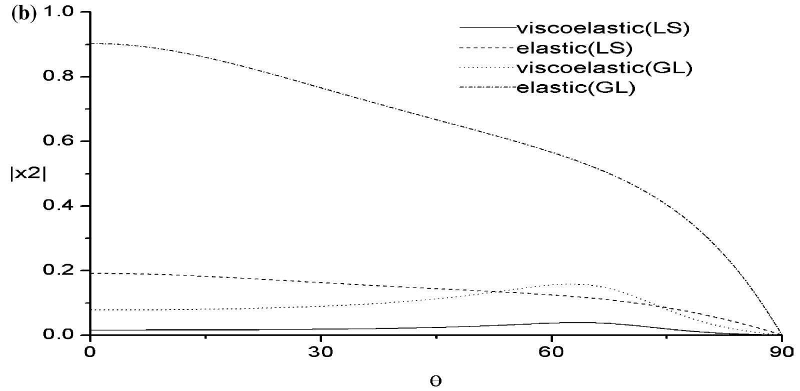


Fig. 4. Viscous effect on variation of reflection coefficient ratio with incident angle of rotational wave.

(a)



(b)



(c)

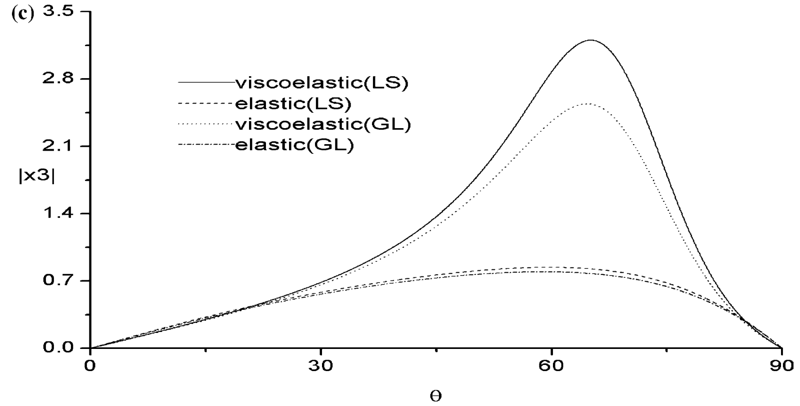


Fig. 5. Viscous effect on variation of reflection coefficient ratio with incident angle of dilatational wave.

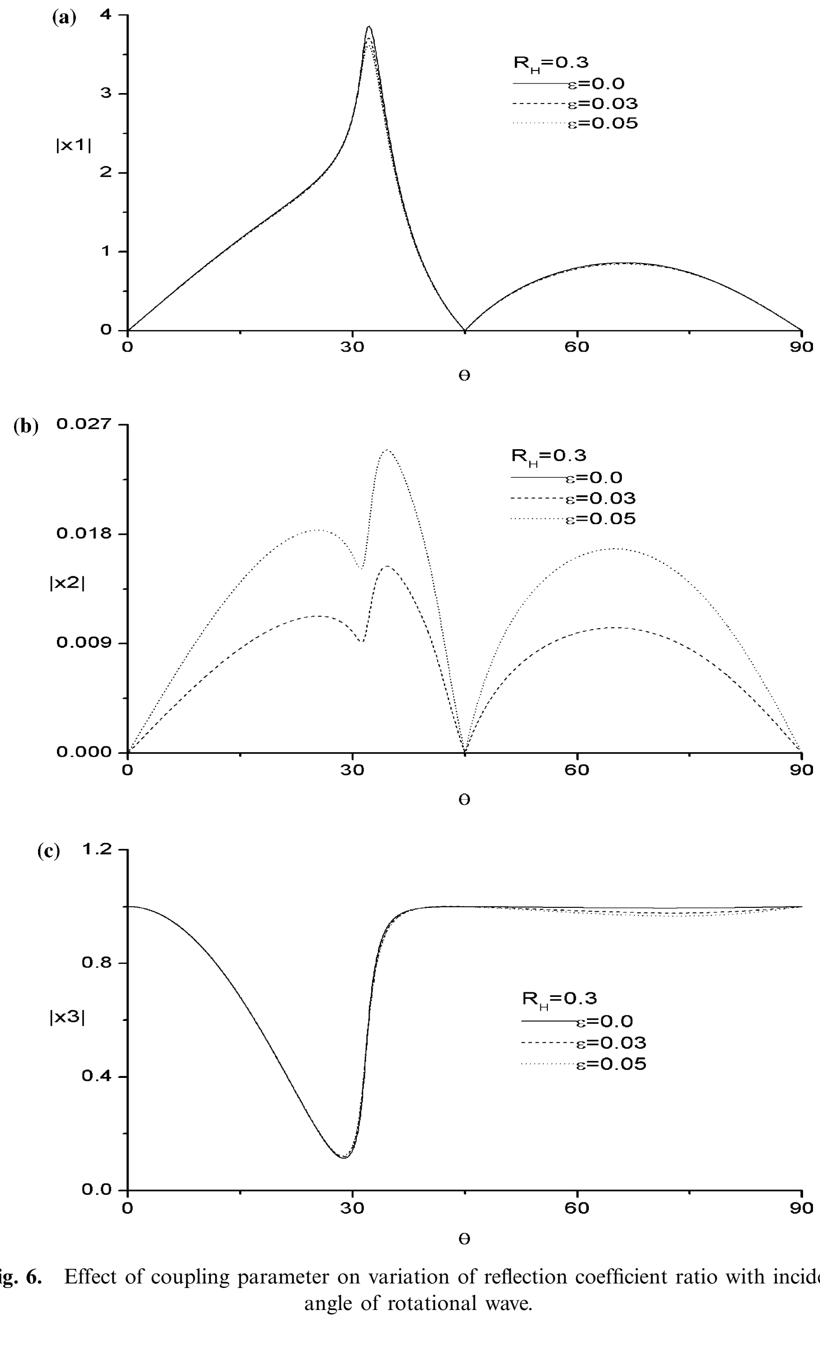


Fig. 6. Effect of coupling parameter on variation of reflection coefficient ratio with incident angle of rotational wave.

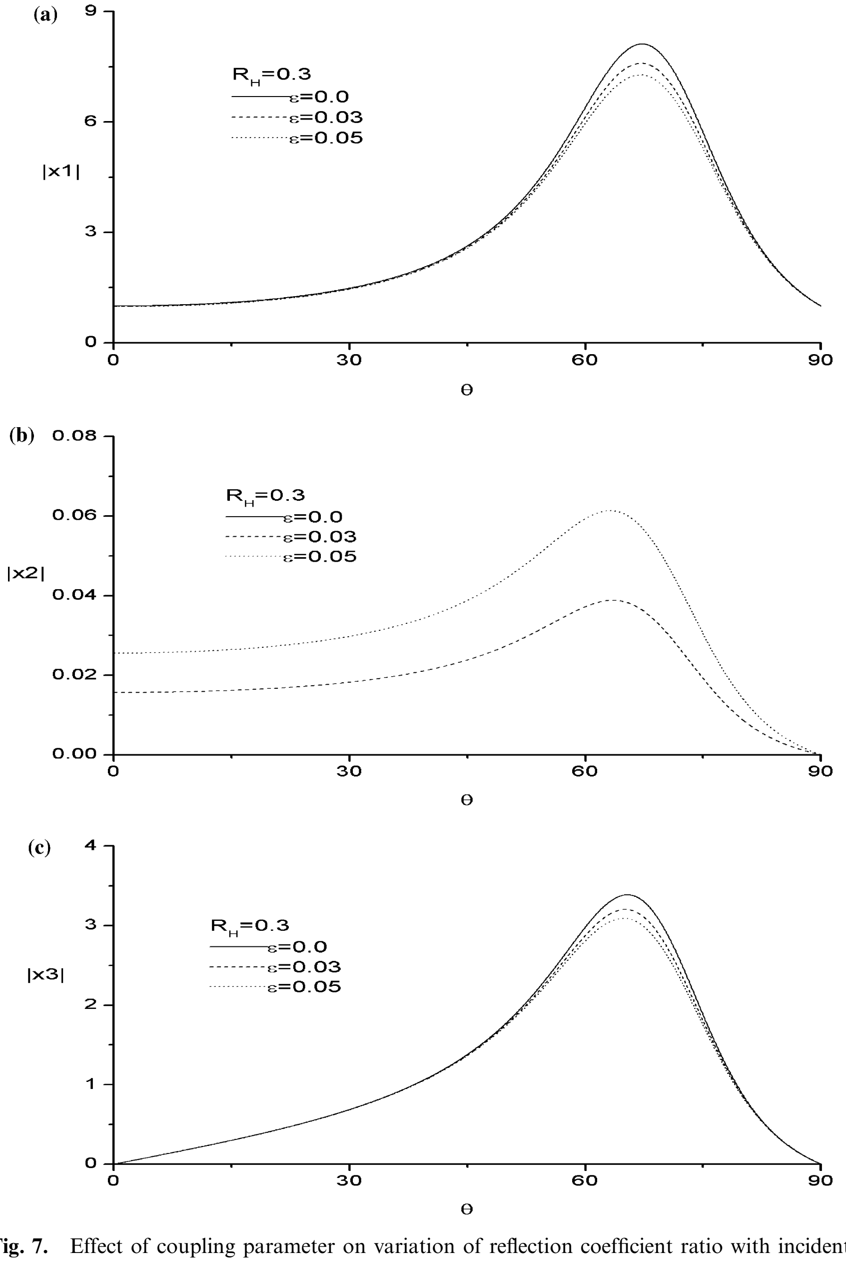


Fig. 7. Effect of coupling parameter on variation of reflection coefficient ratio with incident angle of dilatational wave.

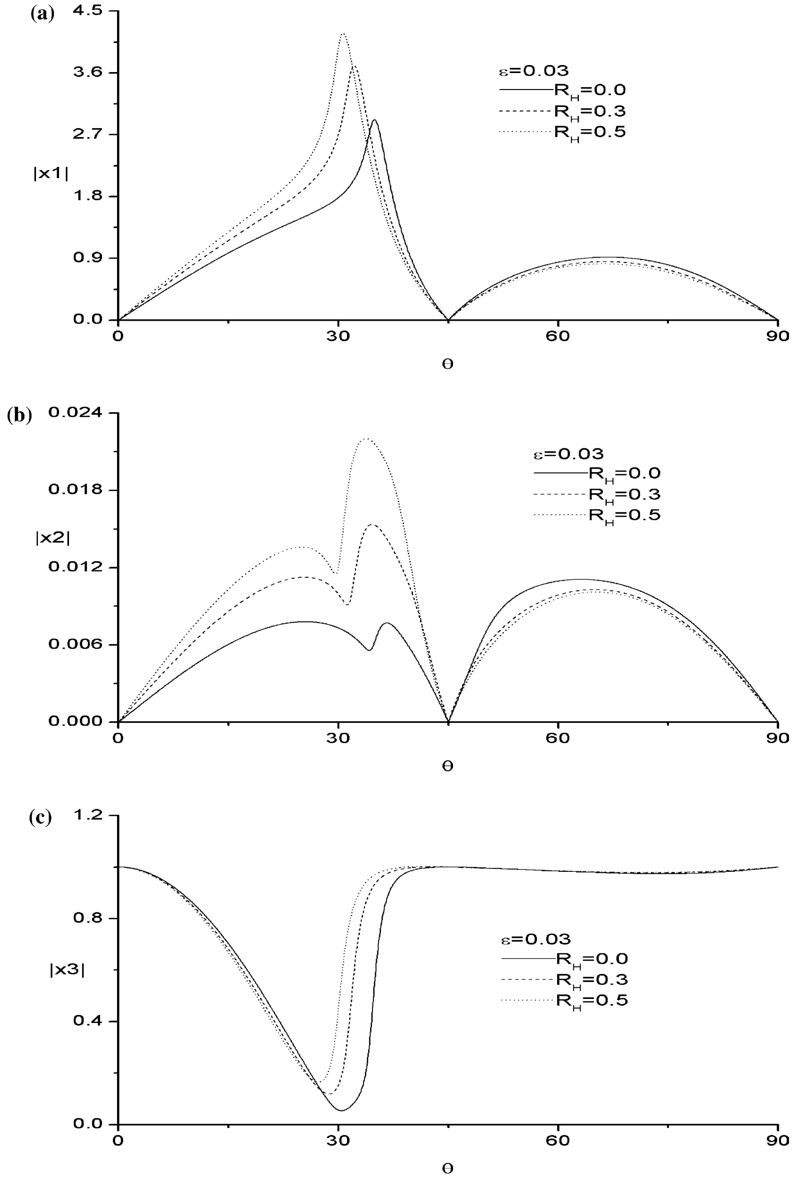


Fig. 8. Effect of magnetic field on variation of reflection coefficient ratio with incident angle of rotational wave.

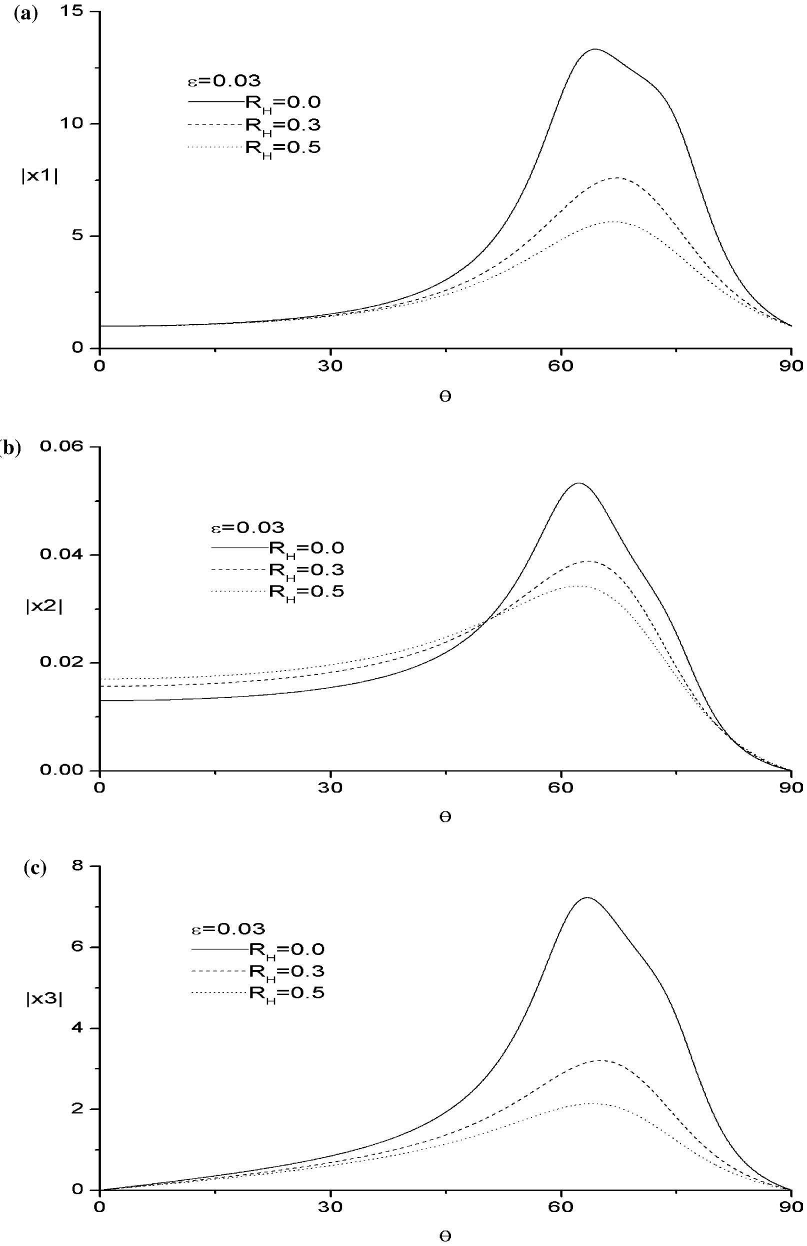


Fig. 9. Effect of magnetic field on variation of reflection coefficient ratio with incident angle of dilatational wave.

2. Viscous effects play a significant role, and the effect is more pronounced in the GL theory than in the LS theory.
3. The thermal coupling parameter and the magnetic field have a salient influence on the reflection coefficient ratio.

ACKNOWLEDGMENTS

This work was supported by grants (Nos. 10132010 and 50175089) from the National Nature Foundation of PRC and the Doctoral Foundation of Xi'an Jiaotong University.

NOMENCLATURE

λ, μ	Lame's constants	ρ	density
C_E	specific heat at constant strain	t	time
T	absolute temperature	T_0	reference temperature
σ_{ij}	components of stress tensor	ε_{ij}	components of strain tensor
u_i	components of displacement vector	k	thermal conductivity
μ_0	magnetic permeability	ε_0	dielectric constant
ρ_e	current density	τ_0, τ_1	relaxation time
e	dilatation	α_t	coefficient of linear thermal expansion
γ	$(3\lambda+2\mu)a_t$	$\vec{\mathbf{H}}$	initial uniform magnetic intensity vector
$\vec{\mathbf{E}}$	induced electric field vector	$\vec{\mathbf{h}}$	induced magnetic field vector
$\vec{\mathbf{u}}$	displacement vector	$\vec{\mathbf{J}}$	current density vector
K	λ+ (2/3)μ Bulk modulus	ε	coupling parameter
$R(t)$	relaxation function	A, β, α^*	empirical constants

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